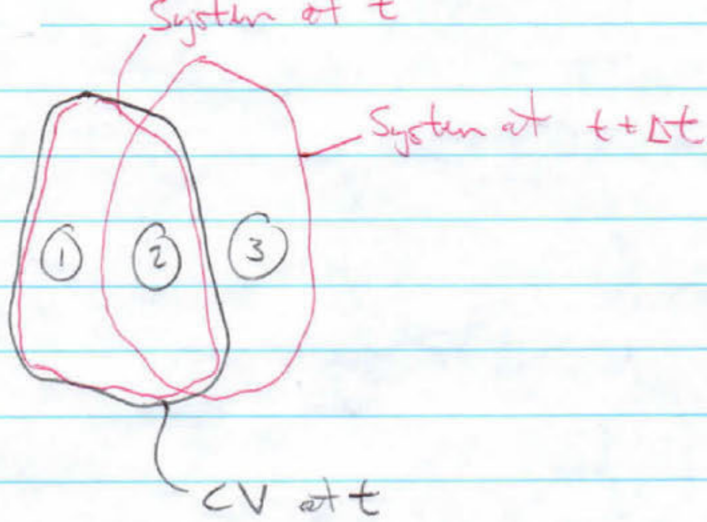
Div Thm of Gauss

$$\frac{\partial}{\partial t}\iiint \rho \, dV + \iint \underline{F} \cdot \hat{n} \, d\sigma = \iiint \nabla \cdot \underline{F} \, dV$$

$$\frac{\partial}{\partial t}\iiint \rho \, dV + \iint \rho V \cdot \hat{n} \, d\sigma = \iiint \nabla \cdot \rho \underline{V} \, dV$$

Reynolds Transport Thm



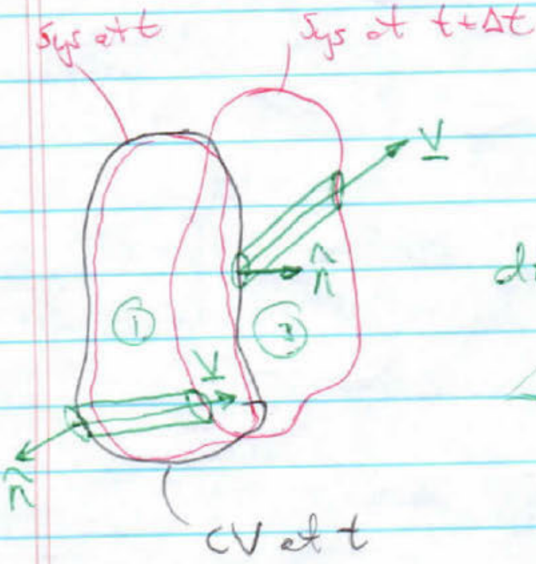
Look at $\dfrac{DN_{sys}}{Dt} = \lim_{\Delta t \to 0} \dfrac{N_{sys}(t+\Delta t) - N_{sys}(t)}{\Delta t}$

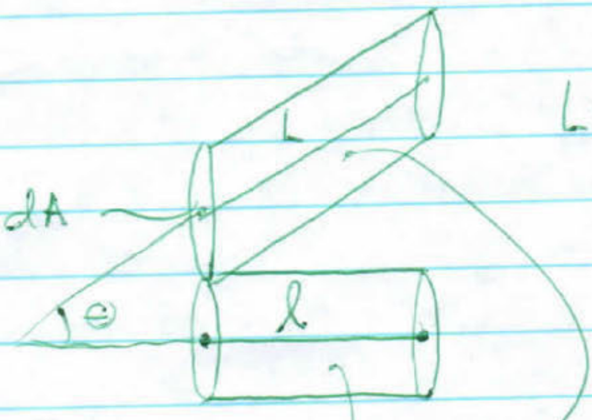
$$= \lim \frac{\left[N_3(t+\Delta t) + N_2(t+\Delta t)\right] - \left[N_2(t) + N_1(t)\right]}{\Delta t}$$

$$= \lim \frac{\left[N_2(t+\Delta t) + \overbrace{N_1(t+\Delta t)}^{*}\right] - \left[N_2(t) + N_1(t)\right] + N_3(t+\Delta t) - \overbrace{N_1(t+\Delta t)}^{*}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{N_{cv}(t+\Delta t) - N_{cv}(t)}{\Delta t} + \lim_{\Delta t \to 0} \frac{N_3(t+\Delta t) - N_1(t+\Delta t)}{\Delta t}$$

$$= \frac{d}{dt} \iiint_{CV} \rho\, \eta \, d\forall + \underbrace{\lim_{\Delta t \to 0} \frac{N_3(t+\Delta t) - N_1(t+\Delta t)}{\Delta t}}_{\text{lets look at this}}$$





$L = |\underline{V}|\,\Delta t$

$$
\begin{aligned}
dV &= dA \cdot l \\
&= dA \cdot L\cos\theta \\
&= dA\,|\underline{V}|\,\Delta t\,\cos\theta \\
&= dA\,\underline{V}\cdot\hat{n}\,\Delta t
\end{aligned}
$$

So $$\frac{N_3(t+\Delta t) - N_1(t+\Delta t)}{\Delta t} = \frac{\iint\limits_{c.\,surf} \rho\eta\,\underline{V}\cdot\hat{n}\,dA\,\Delta t}{\Delta t}$$

$$\therefore \lim_{\Delta t \to 0} \frac{N_3(t+\Delta t) - N_1(t+\Delta t)}{\Delta t} = \iint\limits_{c.s.} \rho\eta\,\underline{V}\cdot\hat{n}\,dA$$

$\underline{V}_{stuff/bndry}$

We are finally left with

Version 1

$$\frac{DN_{sys}}{Dt} = \frac{d}{dt}\iiint_{CV} \rho\eta \, d\forall + \iint_{C.S.} \rho\eta \, \underline{V}_{s/b} \circ \hat{n} \, dA$$

Reynolds Transport Thm.

$\underline{V}$ = velocity of stuff relative to the boundary $= \underline{V}_{r/b}$

Can also write this as (using Leibnitz Thm)

$$\frac{d}{dt}\iiint_{CV} \rho\eta \, d\forall = \iiint_{CV} \frac{\partial}{\partial t}(\rho\eta) \, d\forall + \iint_{C.S.} \rho\eta \, V_{b/o} \circ \hat{n} \, dA$$

in which case RTT becomes

Version 2

$$\frac{D}{Dt} N_{sys} = \iiint_{CV} \frac{\partial}{\partial t}(\rho\eta) \, d\forall + \iint_{CS} \rho\eta \left(\underline{V}_{s/b} + \underline{V}_{b/o}\right) \circ \hat{n} \, dA$$

G.D.T. $\iint \underline{F} \circ \hat{n} \, d\sigma = \iiint \nabla \circ \underline{F} \, dV$

G.R.

$$\frac{\partial}{\partial t} \iiint_R \rho \, dV + \iint_S \rho \underline{V} \circ \hat{n} \, d\sigma = \iiint_R \nabla \circ (\rho \underline{V}) \, dV$$

let $\underline{F} = \rho \underline{V}$

Vect. Identity

$\nabla \circ (\rho \underline{V}) =$

$\rho \nabla \circ \underline{V} + \nabla \rho \circ \underline{V}$